MAY/JUNE 2006

CXC

MATHEMATICS

Paper 02 - General Proficiency

2 hours 40 minutes

LIST OF FORMULAE

Volume of prism length.   V = Ah where A is the area of a cross-section and h is the perpendicular

Volume of cylinder height. $V = \pi r^2 h$ where r is the radius of the base and h is the perpendicular

Volume of a right pyramid height. $V = \frac{1}{3}Ah$ where A is the area of the base and h is the perpendicular

Circumference $C = 2\pi r$ where r is the radius of the circle.

Area of a circle $A = \pi r^2$ where r is the radius of the circle

Area of trapezium the perpendicular height. $A = \frac{1}{2}(a + b)h$ where a and b are the lengths of the parallel sides and h is

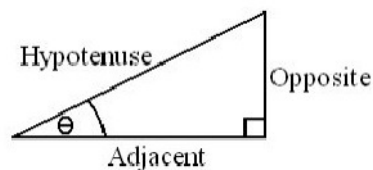Roots of quadratic equations   If $ax^2 + bx + c = 0$,

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometry ratios

$\sin \Theta = \dfrac{\text{opposite side}}{\text{hypotenuse}}$

$\cos \Theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$

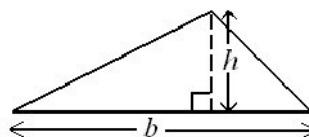$\tan \Theta = \dfrac{\text{opposite side}}{\text{adjacent side}}$

Area of triangle

Area of $\Delta = \frac{1}{2}bh$ where b is the length of the base and h is the perpendicular height.

Area of $\Delta ABC = \frac{1}{2}ab \sin C$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

Where $s = \dfrac{a+b+c}{2}$

Sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

SECTION I

Answer all the questions in this section.

All working must be clearly shown.

1. (a)   Using a calculator, or otherwise, determine the value of $(12.3)^2 - (0.246 \div 3)$ and write the answer

   (i)   exactly

   (ii)   correct to two significant figures

                                                             (2 marks)

   (b)   The table below gives information on the values and the rates of depreciation in value of two motor vehicles.

Motor Vehicle	Initial Value	Yearly Rate of Depreciation	Value after One year
Taxi	$40 000	12%	$p
Private Car	$25 000	q%	$21 250

   Calculate

   (i)   the values of p and q

   (ii)   the value of the taxi after two years.   **(6 marks)**

   (c)   GUY $1.00 = US $0.01 and EC $1.00 = US $0.37

   Calculate the value of

   (i)   GUY $60 000 in US $ **(2 marks)**

   (ii)   US $925 in EC $.   **(2 marks)**

                                                             Total 12 marks

2. (a)   Simplify   **(3 marks)**

   $\dfrac{x-3}{3} - \dfrac{x-2}{5}$

   (b)   (i)   Factorize

        a) $x^2 - 5x$ **(1 mark)**

        b) $x^2 - 81$ **(1 mark)**

     (ii)   Simplify

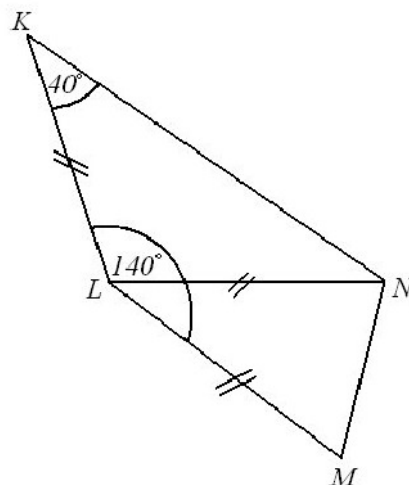 $\dfrac{a^2 + 4a}{a^2 + 3a - 4}$ **(3 marks)**

   (c)   Two cassettes and three CD's cost $175 while four cassettes and one CD cost $125.

(i)   Given that one cassette costs $x and one CD costs $y, write two equations in x and y to represent the information.   **(2 marks)**

(ii)   Calculate the cost of one cassette.   **(2 marks)**

Total 12 marks

3. (a)   In the quadrilateral *KLMN*, **not drawn to scale,** LM = LN = LK, < KLM **=** 140°, and <LKN=40°

Giving the reason for each step of your answer, calculate the size of

(i)   < LNK   **(2 marks)**

(ii)   < NLM   **(2 marks)**

(iii) <KNM.   **(2 marks)**

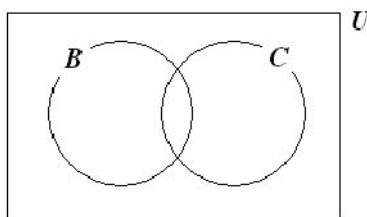(b)   In a survey of 39 students, it was found that

18 can ride a bicycle

15 can drive a car

x can ride a bicycle and drive a car,

3x can do neither.

(i)   Copy and complete the Venn diagram to represent the information.

(ii)   Write an expression in x for the number of students in the survey.

(iii)   Calculate the value of x.

(5 marks)

Total 11 marks

4. (a)   Using a ruler, a pencil and a pair of compasses, construct the triangle ABC in which

AB = 8 cm

< BAC = 60°, and

AC = 5 cm

**(Credit will be given for a neat, clear diagram)   (4 marks)**

(b)   Measure and state the length of BC.   **(1 mark)**

(c)   Find the perimeter of ΔABC.   **(1 mark)**

(d)   Draw on your diagram the line CD which is perpendicular to AB and meets AB at D.   **(2 marks)**

(e)   Determine the length of CD.   **(2 marks)**

(f)   Calculate the area of ΔABC giving your answer to 1 decimal point.   **(2 marks)**

Total 12 marks

5.   The diagram below shows the graph of the function $f(x) = x^2 - 2x - 3$ for $a \leq x \leq b$. The tangent to the graph at (2,3) is also drawn.

Use the graph to determine the

(a)   values of a and b which define the domain of the graph   **(2 marks)**

(b)   values of x for which $x^2 - 2x - 3 = 0$ **(2 marks)**

(c)   coordinates of the minimum point on the graph   **(2 marks)**

(d)   whole number values of x for which $x^2 - 2x - 3 < 1$ **(2 marks)**

(e)   Gradient of $f(x) = x^2 - 2x - 3$ at x = 2.   **(3 marks)**
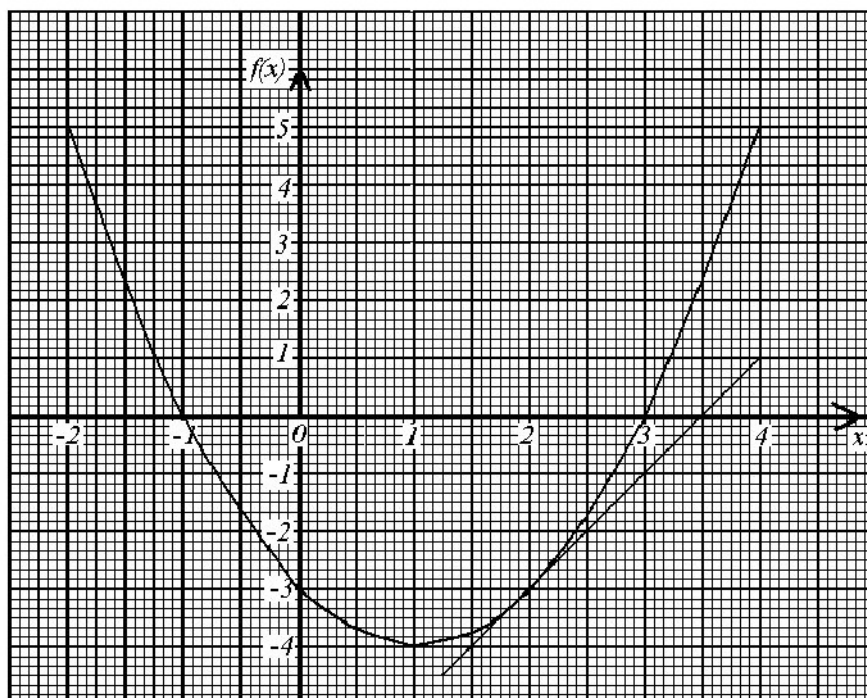
Total 11 marks

6. A man walks x km, due north, from point G to point H. He then walks $(x + 7)$ km due east from H to point F. The distance along a straight line from G to F is 13 km. The diagram below, **not drawn to scale**, shows the relative positions of G, H and F. The direction of north is also shown.
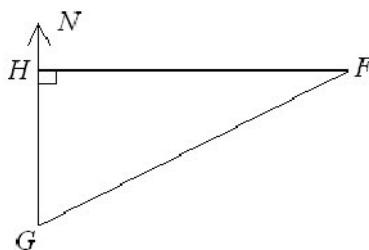
   (a)   Copy the diagram and show on the diagram, the distances x km, $(x + 7)$ km and 13 km.   **(2 marks)**

   (b)   From the information on your diagram, write an equation in x which satisfies Pythagoras' Theorem. Show that the equation can be simplified to give $x^2 + 7x - 60 = 0$ **(3 marks)**

   (c)   Solve the equation and find the distance GH.   **(2 marks)**

   (d)   Determine the bearing of F from G.   **(4 marks)**

Total 11 marks

7.   In an agricultural experiment, the gains in mass, of 100 cows during a certain period were recorded in kilograms as shown in the table below.

Gain in Mass (kg)	Frequency	Mid-Interval Values (kg)
5 - 9	2	7
10 - 14	29	12

15 - 19	37	17
20 - 24	16	
25 -29	14	
30 - 34	2	

(a)   Copy and complete the mid-interval values column.   **(1 mark)**

(b)   (i)   Calculate an estimate of the mean gain in mass of the 100 cows.   **(3 marks)**

   Hint: EACH of the 29 cows in the "10 - 14" interval is assumed to have a mass of 12 kg.

   (ii)   On your answer sheet, complete the drawing of the frequency polygon for the gain in mass of the cows.   **(5 marks)**

(c)   Calculate the probability that a cow chosen at random from the experimental group gained 20 kg or more.   **(2 marks)**

Total 11 marks

8.   The drawings below show a sequence of squares made from toothpicks.
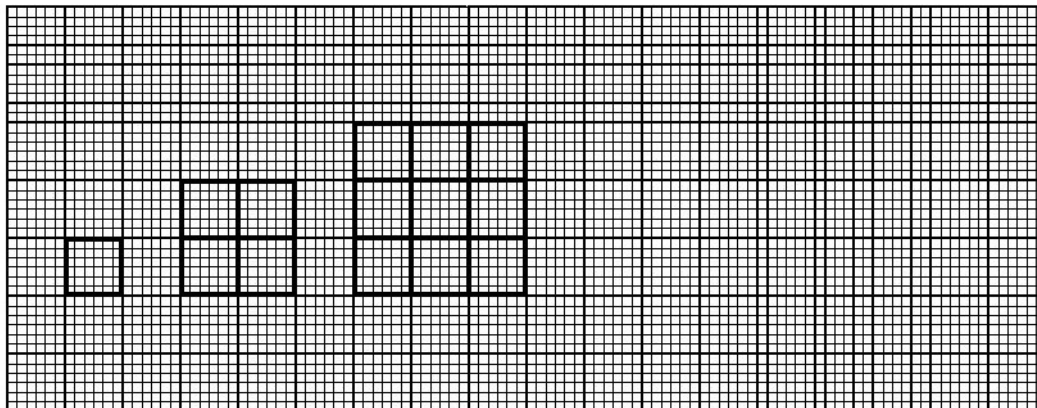
(a)   On the answer sheet provided,

   (i)   draw the next shape in the sequence   **(2 marks)**

   (ii)   insert appropriate values in columns 2 and 3 when

      a)   n = 4

      b)   n = 7   **(4 marks)**

(b)   Complete the tableby inserting appropriate values at

   (i)   r   **(2 marks)**

   (ii)   s   **(2 marks)**

Total 10 marks

SECTION II

Answer TWO questions in this section.

ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

9. (a)   Sove the pair of simultaneous equations
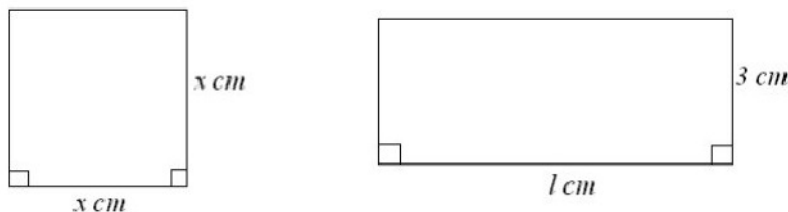
$y = x + 2$

$y = x^2$ **(5 marks)**

(b)   A strip of wire of length 32 cm is cut i nto two pieces. One piece is bent to form a square of side x cm.

The diagrams below, **not drawn to scale**, show the square and the rectangle.

(i)   Write an expression, in terms of l and x, for the lenght of strip of wire.   **(2 marks)**

(ii)   Show that $l = 13 - 2x$ **(2 marks)**

The sum of the areas of the square and the rectangle is represented by S.

(iii)   Show that $S = x^2 - 6x + 39$.   **(2 marks)**

(iv)   Calculate the values of x for which $S = 30.25$.   **(4 marks)**

Total 15 marks

10.   The owner of a parking lot wishes to park x vans and y cars for persons attending a function. The lot provides parking space for no more than 60 vehicles.

(i)   Write an inequality to represent this information.   **(2 marks)**

To get a good bargain, he must provide parking space for at least 10 cars.

(ii)   Write an inequality to represent this information.   **(1 mark)**

The number of cars parked must be fewer than or equal to twice the number of vans parked.

(iii)   Write an inequality to represent this information.   **(2 marks)**

(iv)   (a)   Using a scale of 2 cm to represent 10 vans on the x-axis and 2 cm to represent 10 cars on the y-axis, draw the graphs of the lines associated with the inequalities at (i), (ii) and (iii) above.   **(5 marks)**

(b)   Identify by shading, the region which satisfies **all three** inequalities.   **(1 mark)**

The parking fee for a van is $6 and for a car is $5.

(v)   Write an expression in x and y for the total fees charged for parking x vans and y cars. **(1 mark)**

(vi)   Using your graph write down the coordinates of the vertices of the shaded region.   **(1 mark)**

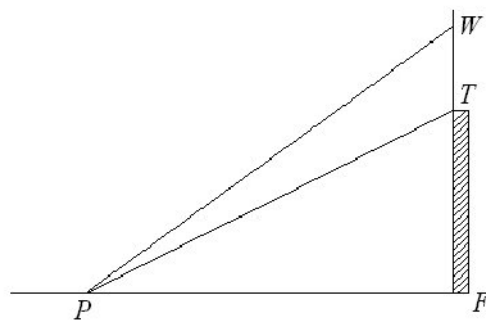(vii)   Calculate the maximum fees charged.   **(2 marks)**

Total 15 marks

GEOMETRY AND TRIGONOMETRY

11. (a)   The diagram below, **not drawn to scale**, shows a vertical tower, FT, and a vertical antenna, TW, mounted on the top of the tower.

A point P is on the same horizontal ground as F, such that PF = 28 m, and the angles of

elevation of T and W from P are 40° and 54° respectively.

    (i)   Copy and label the diagram clearly showing

        a)   the distance 28 m

        b)   the angles of 40° and 54°

        c)   any right angles.

    (ii)   Calculate the length of the antenna TW.

(7 marks)

(b)   The diagram below, **not drawn to scale**, shows a circle, centre O. the lines BD and DCE are tangents to the circle, and angle BCD = 70°
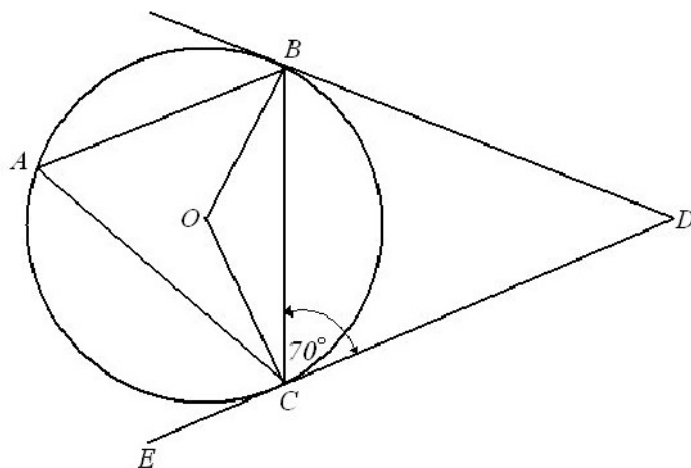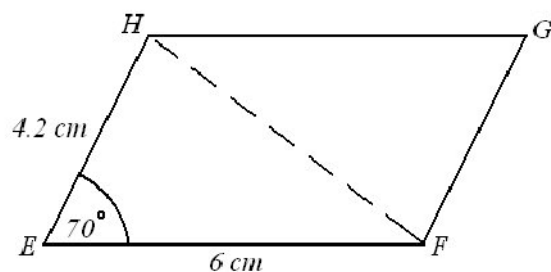

    (i)   <OCE

    (ii)   <BAC

    (iii)   <BOC

    (iv)   <BDC

(8 marks)

Total 15 marks

12. (a)   The diagram below, **not drawn to scale,** shows parallelogram EFGH in which EF = 6 cm, EH = 4.2 cm**,** angle FEH = 70°**.**

Calculate

(i)   the length of the diagonal HF   **(3 marks)**

(ii)   the area of the parallelogram EFGH.   **(2 marks)**

(b)   In this question, use $\pi = 3.14$ and assume the earth to be a sphere of radius 6370 km.

The diagram below, **not drawn to scale**, shows a sketch of the earth with the North ad South poles labelled N and S respectively. The circle of latitude 41°N is shown. Arcs representing circles of longitude 4°E and 74°W are drawn but not labelled.
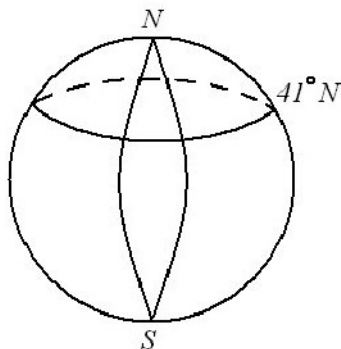

(i)   Copy the sketch above, and draw and label two arcs to represent

a)   the Equator

b)   the Greenwich Meridian.   **(2 marks)**

(ii)   Two points, Y and M, on the surface of the earth have coordinates Y(41°N, 74°W) and M(41°N, 4°E).

a)   Insert the points Y and M on your diagram.   **(2 marks)**

b)   Calculate, correct to the **nearest** kilometre, the circumference of the circle of latitude 41°N.

(3 marks)

c)   Calculate the **shortest** distance between Y and M measured along the circle of latitude 41°N.

(3 marks)

Total 15 marks

VECTORS AND MATRICES

13.   The diagram below shows the position vectors of two points, A and C, relative to an origin, O.
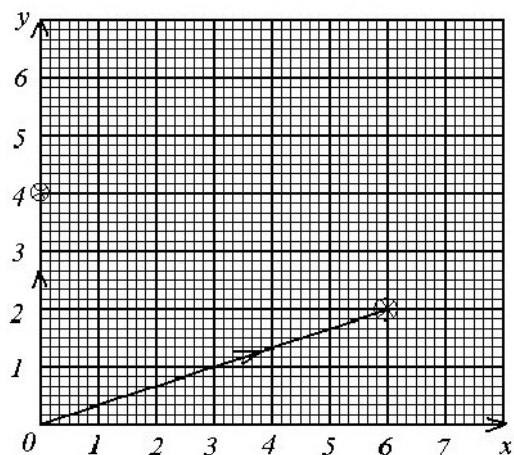
(a)   Copy and complete the diagram to show

     (i)   the point B such that OABC is a parallelogram   **(1 mark)**

     (ii)   the vector $\underline{u}$ where $\underline{u} = \overrightarrow{OA} + \overrightarrow{OC}$ **(2 marks)**

(b)   Write as a column vector, in the form $\begin{pmatrix} x \\ y \end{pmatrix}$, the vector

     (i) $\overrightarrow{OA}$ **(1 mark)**

     (ii) $\overrightarrow{OC}$ **(1 mark)**

     (iii) $\overrightarrow{AC}$ **(2 marks)**

(c)   Given that G is the midpoint of OB, use a vector method to

     (i)   determine the coordinates of G   **(3 marks)**

     (ii)   prove, using a vector method, that A, G, and C lie on a straight line.   **(5 marks)**

                                                   Total 15 marks

14. (a)   The value of determinant of M = $\begin{pmatrix} 2 & 3 \\ -1 & x \end{pmatrix}$ is 9.

     (i)   Calculate the value of x.   **(3 marks)**

     (ii)   For this value of x, find M^{-1}.   **(2 marks)**

     (iii)   Show that $M^{-1}M = I$.   **(2 marks)**

(b)   The graph below shows the line segment AC and its image A'C' after a transformation by the matrix $\begin{pmatrix} p & q \\ r & s \end{pmatrix}$
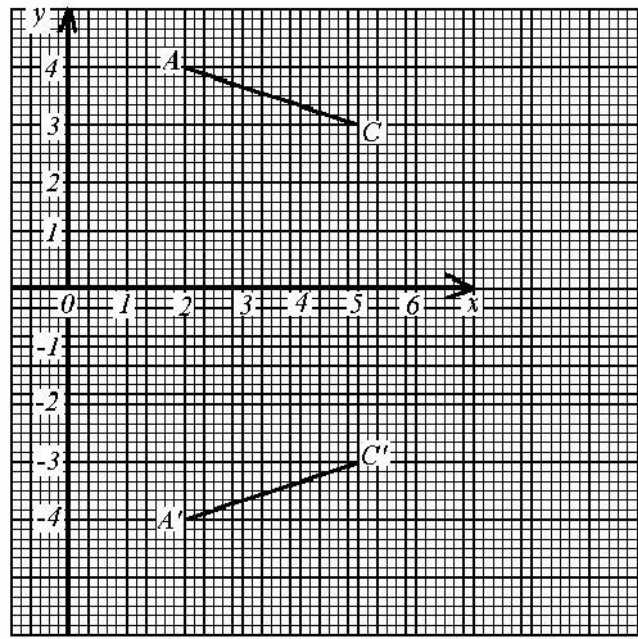
(i)   write in the form of a single 2 × 2 matrix, the coordinates of

    a)   A and C   **(2 marks)**

    b)   A' and C'   **(2 marks)**

(ii)   Using matrices only, write an equation to represent the transformation of AC onto A'C'.
(2marks)

(iii)   Determine the values of p, q, r and s.   **(2 marks)**

Total 15 marks

END OF TEST